$$2\frac{dy}{dx} + 3y = \sin 2x$$

$$\frac{dy}{dx} + \frac{3}{2}y = \frac{1}{2}\sin 2x \quad ; \quad y(0) = 6$$

this is a linear differential equation of first degree first order hence it's integrating factor is

$$I.F. = e^{\int \frac{3}{2} dx} = e^{\frac{3}{2}x}$$

Now it's solution is given by

$$y \cdot e^{3/2 x} = \int \frac{1}{2} \sin 2x \, e^{3/2 x} \, dx + C \quad \text{—(1)}$$

where C is intergrating constant

$$\because \int e^{ax} \sin bx = \frac{e^{ax}}{a^2+b^2}\left[a \sin bx - b\cos bx\right]$$

comparing here $a = \frac{3}{2}$; $b = 2$

So

$$\int e^{3/2 x} \sin 2x = \frac{e^{3/2 x}}{\left(\frac{3}{2}\right)^2 + 2^2}\left[\frac{3}{2}\sin 2x - 2\cos 2x\right]$$

$$= \frac{e^{3/2 x} \times 4}{9+16}\left[\frac{3}{2}\sin 2x - 2\cos 2x\right]$$

$$\int e^{3/2 x} \sin 2x = \frac{4}{25} e^{3/2 x}\left[\frac{3}{2}\sin 2x - 2\cos 2x\right] \quad \text{—(2)}$$

Putting this value of equation (2) in equation (1) we get

$$y \cdot e^{\frac{3x}{2}} = \frac{1}{2} \cdot \frac{4}{25} e^{3/2 x} \left[\frac{3}{2} \sin 2x - 2\cos 2x \right] + C$$

$$y \cdot e^{\frac{3x}{2}} = \frac{2}{25} e^{3/2 x} \left[\frac{3}{2} \sin 2x - 2\cos 2x \right] + C$$

$$y(x) = \frac{2}{25} \left[\frac{3}{2} \sin 2x - 2\cos 2x \right] + C e^{-3/2 x}$$

Now $\because$ $y(0) = 6$ so from this initial condition we find value of constant so

$$y(0) = \frac{2}{25} \left[\frac{3}{2} \sin 2(0) - 2\cos 2(0) \right] + C e^{-\frac{3}{2}(0)}$$

$\because e^0 = 1$

$\cos 0 = 1$

$\sin(0) = 0$

$$6 = \frac{2}{25} \left[0 - 2 \right] + C \cdot 1$$

$$C = 6 + \frac{4}{25} = \frac{150+4}{25} = \frac{154}{25}$$

$$C = \frac{154}{25} = 6.16$$

hence $y(x) = \frac{2}{25} \left[\frac{3}{2} \sin 2x - 2\cos 2x \right] + 6.16 e^{-\frac{3}{2} x}$

$$y(2) = \frac{2}{25} \left[\frac{3}{2} \sin 4 - 2\cos 4 \right] + 6.16 e^{-3}$$

$$y(2) = 0.08 \left[(1.5)(0.06976) - 2(0.99756) \right] + (6.16)(0.049787)$$

$$y(2) = 0.15545$$ Ans