elements

The unit normal vector to $f(x,y,z)$ is defined by

$$n = \frac{\nabla f}{||\nabla f||}$$

$$f(x,y,z) = x^2 + y - z - 4$$; at point $P = (2,0,0)$

$$\nabla f = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(x^2 + y - z - 4)$$

$$\Rightarrow \nabla f = i\frac{\partial}{\partial x}(x^2+y-z-4) + j\frac{\partial}{\partial y}(x^2+y-z-4) + k\frac{\partial}{\partial z}(x^2+y-z-4)$$

$$\Rightarrow \quad = (2x)\hat{i} + \hat{j} - \hat{k}$$

$$\nabla f = (2x)\hat{i} + \hat{j} - \hat{k}$$

at Point $P(2,0,0)$

$$\nabla f_{at\,P} = 2\times 2\hat{i} + \hat{j} - \hat{k}$$

$$= 4\hat{i} + \hat{j} - \hat{k}$$

$$\nabla f = \langle 4, 1, -1\rangle$$

& $$\|\nabla f\| = \sqrt{4^2+1^2+(-1)^2}$$

$$= \sqrt{16+1+1}$$

$$\|\nabla f\| = \sqrt{18}$$

Now $$\vec{n} = \frac{\nabla f}{\|\nabla f\|} = \frac{\langle 4, 1, -1\rangle}{\sqrt{18}}$$

$$\vec{n} = \frac{1}{3\sqrt{2}}\langle 4, 1, -1\rangle$$

$$\vec{n} = \left\langle \frac{4}{3\sqrt{2}}, \frac{1}{3\sqrt{2}}, \frac{-1}{3\sqrt{2}} \right\rangle$$