$$\lim_{x \to 0} \left(x^2 \left(1+2+3+\cdots\left[\frac{1}{|x|}\right]\right)\right)$$

Let $\dfrac{1}{|x|} = h$

Then $$\lim_{h \to \infty} \left(\frac{1+2+3+\cdots[h]}{h^2} \right)$$

Since $h-1 \le [h] \le h$.

$$\Rightarrow \frac{1+2+\cdots h-1}{h^2} \le \frac{1+2+\cdots[h]}{h^2} \le \frac{1+2+\cdots+h}{h^2}$$

$$\Rightarrow \frac{(h-1)h}{2h^2} \le \frac{1+2+\cdots[h]}{h^2} \le \frac{h(h+1)}{2h^2}$$

$$\Rightarrow \lim_{h \to \infty} \frac{h-1}{2h} \le \lim_{h \to \infty} \frac{1+2+\cdots[h]}{h^2} \le \lim_{h \to \infty} \frac{h+1}{2h}$$

$$\Rightarrow \frac{1}{2} \le \lim_{h \to \infty} \frac{1+2+\cdots[h]}{h^2} \le \frac{1}{2}$$

by squeeze theorem.

$$\lim_{h \to \infty} \frac{1+2+\cdots[h]}{h^2} = \lim_{x \to 0} x^2\left(1+2+\cdots\left[\frac{1}{|x|}\right]\right) = \frac{1}{2}$$