Problem:

$$\int x^2 \mathrm{e}^{-x^2}\,\mathrm{d}x$$

Integrate by parts: $\int \mathtt{f}\mathtt{g}' = \mathtt{f}\mathtt{g} - \int \mathtt{f}'\mathtt{g}$

$$\mathtt{f} = x,\quad \mathtt{g}' = x\mathrm{e}^{-x^2}$$

↓ steps ↓ steps

$$\mathtt{f}' = 1,\quad \mathtt{g} = -\frac{\mathrm{e}^{-x^2}}{2}:$$

$$= -\frac{x\mathrm{e}^{-x^2}}{2} - \int -\frac{\mathrm{e}^{-x^2}}{2}\,\mathrm{d}x$$

Now solving:

$$\int -\frac{\mathrm{e}^{-x^2}}{2}\,\mathrm{d}x$$

Apply linearity:

$$= -\frac{\sqrt{\pi}}{4}\int \frac{2\mathrm{e}^{-x^2}}{\sqrt{\pi}}\,\mathrm{d}x$$

Now solving:

$$\int \frac{2\mathrm{e}^{-x^2}}{\sqrt{\pi}}\,\mathrm{d}x$$

This is a special integral (Gauss error function):

$$= \operatorname{erf}(x)$$

Plug in solved integrals:

$$-\frac{\sqrt{\pi}}{4}\int\frac{2e^{-x^2}}{\sqrt{\pi}}\,dx$$

$$=-\frac{\sqrt{\pi}\operatorname{erf}(x)}{4}$$

Plug in solved integrals:

$$-\frac{xe^{-x^2}}{2}-\int-\frac{e^{-x^2}}{2}\,dx$$

$$=\frac{\sqrt{\pi}\operatorname{erf}(x)}{4}-\frac{xe^{-x^2}}{2}$$

The problem is solved:

$$\int x^2e^{-x^2}\,dx$$

$$=\frac{\sqrt{\pi}\operatorname{erf}(x)}{4}-\frac{xe^{-x^2}}{2}+C$$

Rewrite/simplify:

$$=\frac{\sqrt{\pi}\operatorname{erf}(x)-2xe^{-x^2}}{4}+C$$

ANTIDERIVATIVE COMPUTED BY MAXIMA:

$\int f(x)\,dx = F(x) =$

$$\frac{\sqrt{\pi}\operatorname{erf}(x)}{4}-\frac{xe^{-x^2}}{2}+C$$

Simplify

DEFINITE INTEGRAL:

$$\int_0^\infty f(x)\,\mathrm{d}x = $$

$$\frac{\sqrt{\pi}}{4}$$

Approximation:

$$0.443113462726379$$

Simplify

Look up definition: C, e, $\mathrm{erf}(z)$, π