Problem:

$$\int \frac{5x^2}{x^2+4x+3}\,\mathrm{d}x$$

Apply linearity:

$$=5\int \frac{x^2}{x^2+4x+3}\,\mathrm{d}x$$

Now solving:

$$\int \frac{x^2}{x^2+4x+3}\,\mathrm{d}x$$

Perform polynomial long division:

$$=\int\left(\frac{-4x-3}{x^2+4x+3}+1\right)\mathrm{d}x$$

Apply linearity:

$$=\int 1\,\mathrm{d}x-\int \frac{4x+3}{x^2+4x+3}\,\mathrm{d}x$$

Now solving:

$$\int 1\,\mathrm{d}x$$

Apply constant rule:

$$=x$$

Now solving:

$$\int \frac{4x+3}{x^2+4x+3}\,\mathrm{d}x$$

Write $4x+3$ as $2\left(2x+4\right)-5$ and split:

$$=\int\left(\frac{2\left(2x+4\right)}{x^2+4x+3}-\frac{5}{x^2+4x+3}\right)\mathrm{d}x$$

Apply linearity:

$$= 4\int \frac{x+2}{x^2+4x+3}\,\mathrm{d}x - 5\int \frac{1}{x^2+4x+3}\,\mathrm{d}x$$

Now solving:

$$\int \frac{x+2}{x^2+4x+3}\,\mathrm{d}x$$

Substitute $u = x^2+4x+3 \longrightarrow \dfrac{\mathrm{d}u}{\mathrm{d}x} = 2x+4$

(steps) $\longrightarrow \mathrm{d}x = \dfrac{1}{2x+4}\,\mathrm{d}u$:

$$= \int \frac{1}{2u}\,\mathrm{d}u$$

Simplify:

$$= \frac{1}{2}\int \frac{1}{u}\,\mathrm{d}u$$

Now solving:

$$\int \frac{1}{u}\,\mathrm{d}u$$

This is a standard integral:

$$= \ln(u)$$

Plug in solved integrals:

$$\frac{1}{2}\int \frac{1}{u}\,\mathrm{d}u$$

$$= \frac{\ln(u)}{2}$$

Undo substitution $u = x^2+4x+3$:

$$= \frac{\ln\left(x^2+4x+3\right)}{2}$$

Now solving:

$$\int \frac{1}{x^2+4x+3}\,\mathrm{d}x$$

Factor the denominator:

$$=\int \frac{1}{(x+1)(x+3)}\,\mathrm{d}x$$

Perform partial fraction decomposition:

$$=\int \left(\frac{1}{2(x+1)}-\frac{1}{2(x+3)}\right)\mathrm{d}x$$

Apply linearity:

$$=\frac{1}{2}\int \frac{1}{x+1}\,\mathrm{d}x-\frac{1}{2}\int \frac{1}{x+3}\,\mathrm{d}x$$

Now solving:

$$\int \frac{1}{x+1}\,\mathrm{d}x$$

Substitute $u=x+1 \longrightarrow \frac{\mathrm{d}u}{\mathrm{d}x}=1$ (steps) $\longrightarrow$ $\mathrm{d}x=\mathrm{d}u$:

$$=\int \frac{1}{u}\,\mathrm{d}u$$

Use previous result:

$$=\ln(u)$$

Undo substitution $u=x+1$:

$$=\ln(x+1)$$

Now solving:

$$\int \frac{1}{x+3}\,\mathrm{d}x$$

Substitute $u = x + 3 \longrightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 1$ (steps) $\longrightarrow$ $\mathrm{d}x = \mathrm{d}u$:

$$= \int \frac{1}{u}\,\mathrm{d}u$$

Use previous result:

$$= \ln(u)$$

Undo substitution $u = x + 3$:

$$= \ln(x+3)$$

Plug in solved integrals:

$$\frac{1}{2}\int \frac{1}{x+1}\,\mathrm{d}x - \frac{1}{2}\int \frac{1}{x+3}\,\mathrm{d}x$$

$$= \frac{\ln(x+1)}{2} - \frac{\ln(x+3)}{2}$$

Plug in solved integrals:

$$4\int \frac{x+2}{x^2+4x+3}\,\mathrm{d}x - 5\int \frac{1}{x^2+4x+3}\,\mathrm{d}x$$

$$= 2\ln\left(x^2+4x+3\right) + \frac{5\ln(x+3)}{2} - \frac{5\ln(x+1)}{2}$$

Plug in solved integrals:

$$\int 1\,\mathrm{d}x - \int \frac{4x+3}{x^2+4x+3}\,\mathrm{d}x$$

$$= -2\ln\left(x^2+4x+3\right) - \frac{5\ln(x+3)}{2} + \frac{5\ln(x+}{2}$$

Plug in solved integrals:

$$5\int\frac{x^2}{x^2+4x+3}\,\mathrm{d}x$$

$$= -10\ln\left(x^2+4x+3\right)-\frac{25\ln(x+3)}{2}+\frac{25\ln(x}{2}$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int\frac{5x^2}{x^2+4x+3}\,\mathrm{d}x$$

$$= -10\ln\left(\left|x^2+4x+3\right|\right)-\frac{25\ln(|x+3|)}{2}+\frac{25\ln}{}$$

Rewrite/simplify:

$$= -\frac{5\left(4\ln(|x+1|\,|x+3|)+5\ln(|x+3|)-5\ln(\right.}{2}$$

ANTIDERIVATIVE COMPUTED BY MAXIMA:

$\int f(x)\,\mathrm{d}x = F(x) =$

$$5\left(-\frac{9\ln(|x+3|)}{2}+\frac{\ln(|x+1|)}{2}+x\right)+C$$

Simplify/rewrite:

$$5\left(\frac{\ln(|x+1|)-9\ln(|x+3|)}{2}+x\right)+C$$

DEFINITE INTEGRAL:

$\int_1^2 f(x)\,\mathrm{d}x =$

$$5\left(\frac{9\ln(4)-\ln(2)-2}{2}-\frac{9\ln(5)-\ln(3)-4}{2}\right)$$