Problem:

$$\int \arcsin\left(\sqrt{\frac{x}{x+a}}\right) dx$$

Rewrite/simplify:

$$= \int \arcsin\left(\frac{\sqrt{x}}{\sqrt{x+a}}\right) dx$$

Integrate by parts: $\int \mathtt{f}\mathtt{g}' = \mathtt{f}\mathtt{g} - \int \mathtt{f}'\mathtt{g}$

$$\mathtt{f} = \arcsin\left(\frac{\sqrt{x}}{\sqrt{x+a}}\right), \quad \mathtt{g}' = 1$$

↓ steps ↓ steps

$$\mathtt{f}' = \frac{\frac{1}{2\sqrt{x}\sqrt{x+a}} - \frac{\sqrt{x}}{2(x+a)^{\frac{3}{2}}}}{\sqrt{1-\frac{x}{x+a}}}, \quad \mathtt{g} = x:$$

$$= x\arcsin\left(\frac{\sqrt{x}}{\sqrt{x+a}}\right) - \int \frac{x\left(\frac{1}{2\sqrt{x}\sqrt{x+a}} - \frac{\sqrt{x}}{2(x+a)^{\frac{3}{2}}}\right)}{\sqrt{1-\frac{x}{x+a}}}$$

Now solving:

$$\int \frac{x\left(\frac{1}{2\sqrt{x}\sqrt{x+a}} - \frac{\sqrt{x}}{2(x+a)^{\frac{3}{2}}}\right)}{\sqrt{1-\frac{x}{x+a}}} dx$$

Apply linearity:

$$= \frac{1}{2}\int \frac{x\left(\frac{1}{\sqrt{x}\sqrt{x+a}} - \frac{\sqrt{x}}{(x+a)^{\frac{3}{2}}}\right)}{\sqrt{1-\frac{x}{x+a}}} dx$$

Now solving:

$$\int \frac{x\left(\frac{1}{\sqrt{x}\sqrt{x+a}} - \frac{\sqrt{x}}{(x+a)^{\frac{3}{2}}}\right)}{\sqrt{1-\frac{x}{x+a}}}\,\mathrm{d}x$$

Put terms over a common denominator:

$$= \int \frac{\sqrt{a}\sqrt{x}}{x+a}\,\mathrm{d}x$$

Apply linearity:

$$= \sqrt{a}\int \frac{\sqrt{x}}{x+a}\,\mathrm{d}x$$

Now solving:

$$\int \frac{\sqrt{x}}{x+a}\,\mathrm{d}x$$

Substitute $u = \sqrt{x} \longrightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{2\sqrt{x}}$ (steps) $\longrightarrow$ $\mathrm{d}x = 2\sqrt{x}\,\mathrm{d}u$, use:

$$x = u^2$$

$$= 2\int \frac{u^2}{u^2+a}\,\mathrm{d}u$$

Now solving:

$$\int \frac{u^2}{u^2+a}\,\mathrm{d}u$$

Write u^2 as $u^2 + a - a$ and split:

$$= \int \left(\frac{u^2+a}{u^2+a} - \frac{a}{u^2+a}\right)\mathrm{d}u$$

$$= \int \left(1 - \frac{a}{u^2+a}\right)\mathrm{d}u$$

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Now solving:

$$\int 1 \, du$$

Apply constant rule:

$$= u$$

Now solving:

$$\int \frac{1}{u^2 + a} \, du$$

Substitute $v = \frac{u}{\sqrt{a}} \longrightarrow \frac{dv}{du} = \frac{1}{\sqrt{a}}$ (steps) $\longrightarrow$ $du = \sqrt{a} \, dv$:

$$= \int \frac{\sqrt{a}}{av^2 + a} \, dv$$

Simplify:

$$= \frac{1}{\sqrt{a}} \int \frac{1}{v^2 + 1} \, dv$$

Now solving:

$$\int \frac{1}{v^2 + 1} \, dv$$

This is a standard integral:

$$= \arctan(v)$$

Plug in solved integrals:

$$\frac{1}{\sqrt{a}} \int \frac{1}{v^2 + 1} \, dv$$

$$= \frac{\arctan(v)}{\sqrt{a}}$$

Undo substitution $v = \frac{u}{\sqrt{a}}$:

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$$= \frac{\arctan\left(\frac{u}{\sqrt{a}}\right)}{\sqrt{a}}$$

Plug in solved integrals:

$$\int 1 \,\mathrm{d}u - a \int \frac{1}{u^2 + a} \,\mathrm{d}u$$

$$= u - \sqrt{a} \arctan\left(\frac{u}{\sqrt{a}}\right)$$

Plug in solved integrals:

$$2 \int \frac{u^2}{u^2 + a} \,\mathrm{d}u$$

$$= 2u - 2\sqrt{a} \arctan\left(\frac{u}{\sqrt{a}}\right)$$

Undo substitution $u = \sqrt{x}$:

$$= 2\sqrt{x} - 2\sqrt{a} \arctan\left(\frac{\sqrt{x}}{\sqrt{a}}\right)$$

Plug in solved integrals:

$$\sqrt{a} \int \frac{\sqrt{x}}{x + a} \,\mathrm{d}x$$

$$= 2\sqrt{a}\sqrt{x} - 2a \arctan\left(\frac{\sqrt{x}}{\sqrt{a}}\right)$$

Plug in solved integrals:

$$\frac{1}{2} \int \frac{x\left(\frac{1}{\sqrt{x}\sqrt{x+a}} - \frac{\sqrt{x}}{(x+a)^{\frac{3}{2}}}\right)}{\sqrt{1 - \frac{x}{x+a}}} \,\mathrm{d}x$$

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Plug in solved integrals:

$$\frac{1}{2}\int \frac{x\left(\frac{1}{\sqrt{x}\sqrt{x+a}}-\frac{\sqrt{x}}{(x+a)^{\frac{3}{2}}}\right)}{\sqrt{1-\frac{x}{x+a}}}\,\mathrm{d}x$$

$$=\sqrt{a}\sqrt{x}-a\arctan\left(\frac{\sqrt{x}}{\sqrt{a}}\right)$$

Plug in solved integrals:

$$x\arcsin\left(\frac{\sqrt{x}}{\sqrt{x+a}}\right)-\int \frac{x\left(\frac{1}{2\sqrt{x}\sqrt{x+a}}-\frac{\sqrt{x}}{2(x+a)^{\frac{3}{2}}}\right)}{\sqrt{1-\frac{x}{x+a}}}$$

$$=x\arcsin\left(\frac{\sqrt{x}}{\sqrt{x+a}}\right)-\sqrt{a}\sqrt{x}+a\arctan\left(\frac{\sqrt{x}}{\sqrt{a}}\right)$$

The problem is solved:

$$\int \arcsin\left(\frac{\sqrt{x}}{\sqrt{x+a}}\right)\mathrm{d}x$$

$$=x\arcsin\left(\frac{\sqrt{x}}{\sqrt{x+a}}\right)-\sqrt{a}\sqrt{x}+a\arctan\left(\frac{\sqrt{x}}{\sqrt{a}}\right)$$

ANTIDERIVATIVE COMPUTED BY MAXIMA:

$\int \boldsymbol{f(x)}\,\mathbf{d}\boldsymbol{x} = \boldsymbol{F(x)} =$