

$$2x^2 \frac{d^2 y}{dx^2} + 3x \frac{dy}{dx} - y = 0$$

$$\Rightarrow x^2 \frac{d^2 y}{dx^2} + \frac{3}{2} x \frac{dy}{dx} - \frac{y}{2} = 0$$

Cauchy euler equation

put $x = e^z$

then D.E will become

$$\frac{d^2 y}{dz^2} + \frac{1}{2} \frac{dy}{dz} - \frac{y}{2} = 0$$

A.E

$$m^2 + \frac{m}{2} - \frac{1}{2} = 0$$

$$2m^2 + m - 1 = 0$$

$$2m^2 + 2m - m - 1 = 0$$

$$2m(m+1) - 1(m+1) = 0$$

$$m = -1, \frac{1}{2}$$

Then

$$y(z) = C_1 e^{-z} + C_2 e^{\frac{z}{2}}$$

put $z = \log x$

$$y(x) = C_1 e^{-\log x} + C_2 e^{\frac{\log x}{2}}$$

$$= \boxed{\frac{C_1}{x} + C_2 \sqrt{x}}$$

Given - passes through (1,1)

$$1 = C_1 + C_2 \Rightarrow C_2 = 1 - C_1$$

Then $y(x) = \boxed{\frac{C_1}{x} + (1 - C_1)\sqrt{x}}$

put $C_1 = 1$
 $y = \frac{1}{x}$ is solution
option (A)