

Let R be ROC of $\sum a_n x^n$

$$\Rightarrow \boxed{\frac{1}{R} = \lim_{n \rightarrow \infty} (a_n)^{\frac{1}{n}}} \quad (i)$$

Now

to find ROC of $\sum a_n x^{kn}$

$$\frac{1}{R_1} = \lim_{n \rightarrow \infty} (a_n)^{\frac{1}{kn}} = \lim_{n \rightarrow \infty} \left((a_n)^{\frac{1}{n}} \right)^{\frac{1}{k}}$$

$$= \left(\frac{1}{R} \right)^{\frac{1}{k}} \quad [\text{from (i)}]$$

$$\Rightarrow \boxed{R_1 = (R)^{\frac{1}{k}}}$$