

Given :- $x, x+e^x, 1+x+e^x$ are solutions of D.E

Degree - 2, no. of l.i solution = 3

$\Rightarrow \exists$ Particular integral

$\Rightarrow$ P.I must be present in all solutions of D.E

$\Rightarrow$ here, we can see 'x' is present in all solution.

$\Rightarrow$ 'x' is particular integral.

Then

$$\Rightarrow \boxed{y = C_1 e^{0x} + C_2 e^x + x} \rightarrow \text{C.F. + P.I.}$$

$$\text{or } \boxed{y = C_1 + C_2 e^x + x}$$

$$y(0) = 3 \text{ gives } C_1 + C_2 = 3$$

$$y'(0) = 4 \text{ gives } C_2 e^0 + 1 = 4 \Rightarrow C_2 = 3$$

$$\Rightarrow \boxed{C_2 = 3, C_1 = 0}$$

$$\text{Then } y = 3e^x + x$$

$$\boxed{y(1) = 3e + 1} \quad \checkmark$$