

Formula $\rightarrow$ power series of $f(x)$ at $x=0$ is

$$f(x) = \sum_{n=0}^{\infty} \frac{f^n(0) x^n}{n!}$$

(i)

$$f(x) = \frac{1}{(8+x)^2}$$

$$f(0) = \frac{1}{(8)^2} = \frac{(-1)^0}{64}$$

$$f'(x) = \frac{-2}{(8+x)^3} \rightarrow f'(0) = \frac{-2(-1)^1}{8^3} = \boxed{\frac{(-1)^1(-2)}{8^{1+2}}}$$

$$f''(x) = \frac{-2 \times -3}{(8+x)^4} \rightarrow f''(0) = \frac{6}{8^4} = \frac{3!(-1)^2}{8^4} = \boxed{\frac{3!(-1)^2}{8^{2+2}}}$$

$$f'''(0) = \frac{-24}{(8+x)^5} \rightarrow f'''(0) = \frac{-24}{8^5} = \frac{4!(-1)^3}{8^5} = \boxed{\frac{4!(-1)^3}{8^{3+2}}}$$

$\vdots$

$$f^{(n)}(0) = \frac{(-1)^n (n+1)!}{(8)^{n+2}}$$

from (i)

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (n+1)! x^n}{8^{n+2}}$$

option (C)