$$f(x) = \frac{x}{1+x}$$

we will use <u>taylor series</u>

Since

$$f(x-a+a) = f(x) = f(a) + \frac{f'(a)}{1!}(x-a)^1 + \frac{f''(a)}{2!}(x-a)^2 + \cdots$$

~~to~~ expand the taylor series of $f(x)$ about $\underline{x=2}$

$$f(2) = \frac{2}{1+2} = \underline{\frac{2}{3}}$$

$$f'(x) = \frac{(1+x)-x}{(1+x)^2} = \frac{1}{(1+x)^2}$$

$$f'(2) = \underline{\frac{1}{9}}$$

$$f''(x) = \frac{-2}{(x+1)^3} \Rightarrow f''(2) = \frac{-2}{27}$$

$$\Rightarrow f(x) = f(2) + \frac{f'(2)}{1}(x-2) + \frac{f''(2)}{2}(x-2)^2$$

$$= \frac{2}{3} + \frac{(x-2)}{9} + \frac{(-2)}{27\times 2}(x-2)^2$$

$$\frac{2}{3} + \frac{1}{9}(x-2) + \frac{c(x-2)^2}{(1+\xi)^3} = \frac{2}{3} + \frac{(x-2)}{9} + \frac{-\cancel{2}(x-2)^2}{27\cdot\cancel{2}}$$

$$\frac{2}{3} + \frac{1(x-2)}{9} + \frac{c(x-2)^2}{(1+\xi)^3} = \frac{2}{3} + \frac{(x-2)}{9} + \frac{(-1)(x-2)^2}{(3)^3}$$

On comparing

$$\frac{c}{(1+\xi)^3} = \frac{-1}{3^3}$$

$\boxed{c = -1}$ $\boxed{\xi = 3}$

also $\xi \in (2, x)$