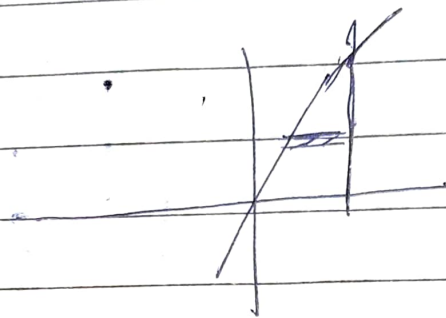


$$\int_0^a \int_0^x \frac{\phi'(y) dy dx}{\sqrt{(a-x)(x-y)}}$$



$$\Rightarrow \int_{y=0}^a \int_{x=y}^a \frac{\phi'(y)}{\sqrt{(a-x)(x-y)}} dx dy$$

$$\Rightarrow \int_{y=0}^a \phi'(y) \left(\int_{x=y}^a \frac{1}{\sqrt{(a-x)(x-y)}} dx \right) dy$$

$I = \text{let}$

$$I = \int_{x=y}^a \frac{1}{\sqrt{(a-x)(x-y)}} dx$$

$$\text{let } a-x = t^2 \Rightarrow -2t dt = dx$$

$$I = \int_{x=y}^a \frac{1}{\sqrt{t^2(a-t^2-y)}} (-2t dt) \Rightarrow -2 \int \frac{dt}{\sqrt{a-y-t^2}}$$

$$\Rightarrow -2 \left[\sin^{-1} \frac{t}{\sqrt{a-y}} \right]_{x=y}^{x=a} \Rightarrow -2 \left[\sin^{-1} \sqrt{\frac{a-x}{a-y}} \right]_{x=y}^{x=a}$$

$$\Rightarrow -2 \left[0 - \frac{\pi}{2} \right] = \underline{\underline{\pi}}$$

from (*)

$$\pi \int_{y=0}^a \phi'(y) dy \Rightarrow \pi [\phi(y)]_0^a$$

$$\Rightarrow \underline{\underline{\pi [\phi(a) - \phi(0)]}}$$