here

$$a_n = \frac{1}{n}[n\beta] + n^2\beta^n \qquad \beta \in (0,1)$$

$$\left[\text{we know } [x] = x - \underbrace{\{x\}}_{\text{fractional part}} \quad \forall x \in R\right]$$

$$\Rightarrow a_n = \frac{1}{n}\left[n\beta - \{n\beta\}\right] + n^2\beta^n$$

bounded

$$\Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty} \beta - \lim_{n\to\infty} \frac{\{n\beta\}}{n} + \lim_{n\to\infty} n^2\beta^n$$

$$= \beta - 0 + \lim_{n\to\infty} n^2\beta^n$$

$= \beta - 0 + 0$

$= \boxed{\beta}$

Thus a_n is convergent to β

let $a = \frac{1}{\beta} \Rightarrow a > 1$

$$\lim_{n\to\infty} n^2\beta^n = \lim_{n\to\infty} \frac{n^2}{a^n} \quad \left(\frac{\infty}{\infty}\right) \text{ case}$$

use l'hopital

$$\lim_{n\to\infty} \frac{2n}{a^n \log a} = \lim_{n\to\infty} \frac{2}{a^n (\log a)^2}$$

$= \boxed{0}$

$$\Rightarrow \boxed{\lim_{n\to\infty} n^2\beta^n = 0}$$