

$$I = \int_{x=0}^{\infty} \int_{y=1+x}^{\infty} e^{\left(-\frac{y}{x+1} - x\right)} dy dx$$

$$\Rightarrow \int_{x=0}^{\infty} e^{-x} \times \left[e^{-\frac{y}{x+1}} - (x+1) \right]_{1+x}^{\infty} dx$$

$$\Rightarrow \int_{x=0}^{\infty} -e^{-x}(x+1) \left[0 - e^{-\frac{x+1}{x+1}} \right] dx$$

$$\Rightarrow \int_{x=0}^{\infty} e^{-(x+1)} (x+1) dx$$

let $x+1=t \Rightarrow dt=dx$

$$\Rightarrow \int t e^{-t} dt \Rightarrow -t e^{-t} - \int 1 \cdot e^{-t} dt$$

$$\Rightarrow \boxed{-t e^{-t} - e^{-t}}$$

$$\Rightarrow \left[-(x+1) e^{-(x+1)} - e^{-(x+1)} \right]_0^{\infty}$$

$$\Rightarrow \left[e^{-(x+1)} [-x-1-1] \right]_0^{\infty}$$

$$\Rightarrow \left[\frac{-x-2}{e^{-(x+1)}} \right]_0^{\infty} \Rightarrow 0 - \left(\frac{-0-2}{e} \right)$$

$$\Rightarrow \boxed{\frac{2}{e}}$$