$$G = \left(\left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \;\middle|\; a,b,c,d \in \mathbb{Z} \right\}, + \right)$$

$$H = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in G \;\middle|\; a+b+c+d=0 \right\}$$

① clearly $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \in H$ as $0+0+0+0=0$

$\Rightarrow$ identity $\in H$ $\Rightarrow$ $\boxed{H \neq \phi}$

② let $x = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ & $y = \begin{bmatrix} a' & b' \\ c' & d' \end{bmatrix} \in H$

$\Rightarrow a+b+c+d = 0 = a'+b'+c'+d'$

then $x+y = \begin{bmatrix} a+a' & b+b' \\ c+c' & d+d' \end{bmatrix} \in H$

as $a+a'+b+b'+c+c'+d+d' =$
$= a+b+c+d+a'+b'+c'+d' = 0+0 = \boxed{0}$

Closure satisfied

③ let $x = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in H \Rightarrow a+b+c+d=0$

then $x^{-1} = \begin{bmatrix} -a & -b \\ -c & -d \end{bmatrix} \in H$

since $-a-b-c-d = -(a+b+c+d) = -0 = \boxed{0}$

Thus by two step test,

H is subgroup of G

part ② if 0 is replaced by 1

Then $H = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| \; a+b+c+d=1 \right\}$

here identity $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \notin H$

as $[0+0+0+0 \neq 1]$

So, H is not subgroup of G