Since $f:[0,1] \to R$ is continuous function

Consider $\int_0^x f(t)\,dt = \int_x^1 f(t)\,dt$

Differentiate both sides wrt x

$$f(x) = f(1)\cdot 0 - f(x)\cdot 1$$

$$2f(x) = 0$$

$$\boxed{f(x) = 0}$$