Consider $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2-y^2}\,dx\,dy = \left(\int_{-\infty}^{\infty} e^{-x^2}\,dx\right)\times\left(\int_{-\infty}^{\infty} e^{-y^2}\,dy\right)$

let $y = x \Rightarrow dy = dx$

$$= \left(\int_{-\infty}^{\infty} e^{-x^2}\,dx\right)\times\left(\int_{-\infty}^{\infty} e^{-x^2}\,dx\right)$$

$$\Rightarrow \left(2\int_{0}^{\infty} e^{-x^2}\,dx\right)\left(2\int_{0}^{\infty} e^{-x^2}\,dx\right) \quad \left[e^{-x^2} \text{ is even function}\right]$$

$$\Rightarrow 4\left(\int_{0}^{\infty} e^{-x^2}\,dx\right)^2 \quad \text{——— (i)}$$

(ii) $$\lim_{b\to\infty}\int_{-b}^{b}\int_{-b}^{b} e^{-x^2-y^2}\,dx\,dy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2-y^2}\,dx\,dy \quad \text{——— (ii)}$$

from (i) & (ii)

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2-y^2}\,dx\,dy = \lim_{b\to\infty}\int_{-b}^{b}\int_{-b}^{b} e^{-x^2-y^2}\,dx\,dy = 4\left(\int_{0}^{\infty} e^{-x^2}\,dx\right)^2$$