Let define a function

$$\boxed{f(x) = a_0 x + a_1 x^2 + a_2 x^3 + \cdots \quad a_n x^{n+1}}$$

Clearly $f(x)$ is continuous on $[0,1]$

" is differentiable on $(0,1)$

Because $f(x)$ is a polynomial function.

Also, $\boxed{f(0) = 0 = f(1)}$

$f(1) = a_0 + a_1 + a_2 + \cdots \quad a_n = 0$ [given]

Thus, all the three conditions of Roll's theorem are satisfied.

Hence $\exists$ at least one point $c \in (0,1)$ such that

$f'(c) = 0$

$\Rightarrow \cancel{a_0 + 2a_1 x + 3a_2 x^2 + \cdots (n+1) a_n x^n}$

$$a_0 + 2a_1 c + 3a_2 c^2 + \cdots \quad (n+1) a_n c^n = 0$$

Or $\boxed{a_0 + 2a_1 x + 3a_2 x^2 + \cdots \quad (n+1) a_n x^n = 0}$

$x \in (0,1)$