

$$dy (x - 2y^3) - dx (2xy^2 + y) = 0$$

let $x^h y^k \equiv$ Integrating factor

then

$$0 = dy (x^{1+h} y^k - 2x^h y^{3+k}) - dx (2x^{1+h} y^{2+k} + y^{1+k} x^h)$$

$$\frac{\partial M}{\partial y} = -2x^{1+h} (2+k) y^{1+k} - (1+k) y^k x^h \quad (i)$$

$$\frac{\partial N}{\partial x} = (1+h) x^h y^k - 2h x^{h-1} y^{3+k}$$

for D.E to be exact

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

$$\Rightarrow 1+h = -1-k \Rightarrow \boxed{h=0}$$

$$\boxed{k=-2}$$

put h, k in (i)

$$\Rightarrow dy (x y^{-2} - 2y) - dx (2x + y^{-1})$$

Verify

$$\left(\frac{\partial M}{\partial y} = -(-1) y^{-2} = \frac{1}{y^2} = \frac{\partial N}{\partial x} \right) \rightarrow \text{exact}$$

Thus $\underline{\text{I.o.F}} = x^0 y^{-2} = \boxed{\frac{1}{y^2}}$ h