

$$\int_0^1 \prod_{n=1}^{2020} (x+n) \cdot \sum_{k=1}^{2020} \frac{1}{(x+k)} dx$$

$$\Rightarrow \int_0^1 (x+1)(x+2) \dots (x+2020) \left[\frac{1}{x+1} + \frac{1}{x+2} + \dots + \frac{1}{x+2020} \right]$$

$$\Rightarrow \int_0^1 (x+1)(x+2) \dots (x+2020) \left[\frac{(x+2)(x+3) \dots (x+2020) + (x+1)(x+3) \dots (x+2020) + \dots}{(x+1)(x+2)(x+3) \dots (x+2020)} \right]$$

$$\Rightarrow \int_0^1 (x+2)(x+3) \dots (x+2020) + (x+1)(x+3) \dots (x+2020) + \dots dx$$

$$\Rightarrow \int_0^1 \frac{d}{dx} [(x+1)(x+2) \dots (x+2020)] dx$$

$$\Rightarrow \left[(x+1)(x+2) \dots (x+2020) \right]_0^1$$

$$\Rightarrow (2 \cdot 3 \cdot 4 \dots 2021) - (1 \cdot 2 \cdot 3 \cdot 4 \dots 2020)$$

$$\Rightarrow (2021)! - (2020)!$$

$$\Rightarrow (2020)! [2021 - 1] \Rightarrow$$

$$(2020)(2020)! \quad \checkmark$$