The function $\sin(1/x)$.

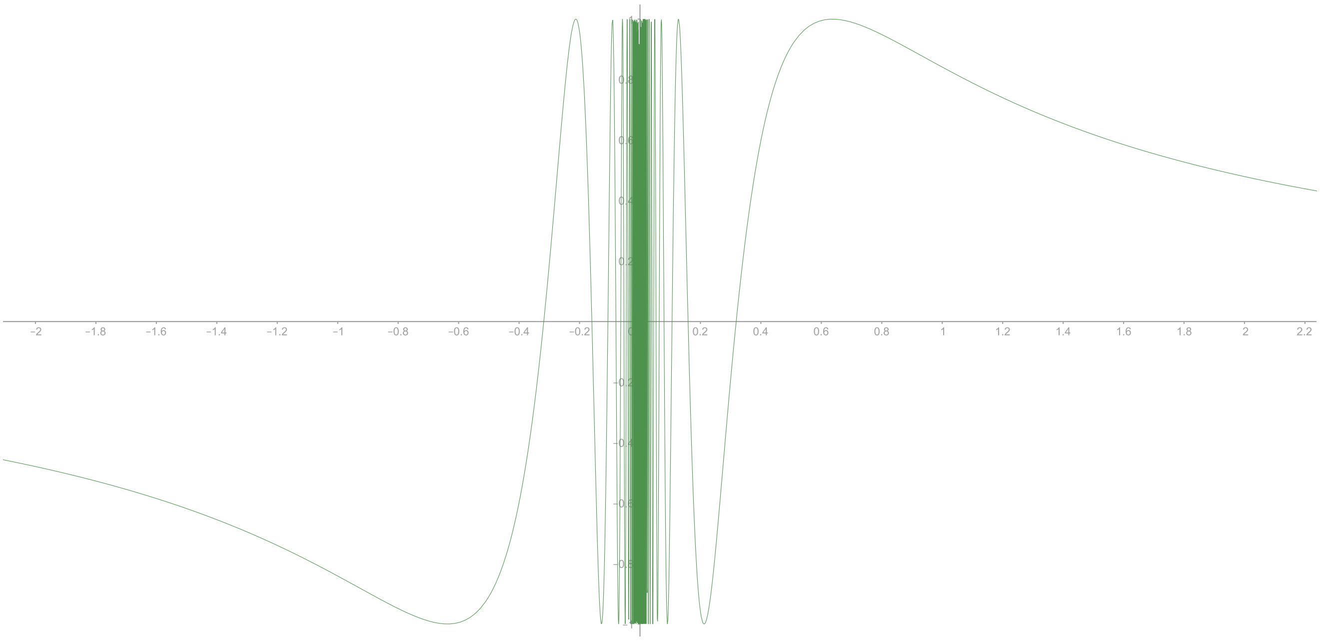
The function $f(x) = \sin(1/x)$ is defined for all $x \neq 0$.

The value $f(0)$ is not defined.

For every $x \neq 0$, $-1 \leq f(x) \leq 1$.

The function is odd, $f(-x) = -f(x)$, its graph is symmetric with respect to the origin $(0,\ 0)$.

As a composition of $1/x$ and $\sin x$, $f(x)$ is continuous at each point of its domain.



As x increases in magnitude, the values of $f(x)$ tend to 0 :

$$\lim_{x\to\infty} \sin(1/x) = \sin(0^+) = 0^+,$$

$$\lim_{x\to-\infty} \sin(1/x) = \sin(0^-) = 0^-.$$

As x approaches 0, the frequency of oscillation increases without bound, while the amplitude stays at a constant level, 1. Therefore the limit of $\sin(1/x)$ at the origin does not exist. In fact, both one-sided limits

$$\lim_{x\to 0^+} \sin(1/x) = \lim_{t\to\infty} \sin t,$$

$$\lim_{x\to 0^-} \sin(1/x) = \lim_{t\to-\infty} \sin t$$

fail to exist. The discontinuity of $\sin(1/x)$ at the origin is essential.