

Result - Cesaro's Theorem

$$\text{If } \lim a_n = a, \quad \lim b_n = b$$

Then

$$\lim_{n \rightarrow \infty} \frac{1}{n} [a_n b_1 + a_{n-1} b_2 + \dots + a_1 b_n] = a \cdot b$$

Now in our Question

$$\lim_{n \rightarrow \infty} \frac{1}{n} \left[\left(\frac{1}{2}\right)^n + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{n-1} + \left(\frac{1}{3}\right)\left(\frac{1}{2}\right)^{n-2} + \dots + \frac{1}{n} \left(\frac{1}{2}\right) \right]$$

$$\text{here } a_n = \left(\frac{1}{2}\right)^n \quad \& \quad b_n = \frac{1}{n}$$

$$\lim_{n \rightarrow \infty} a_n = 0 = a \quad \& \quad \lim_{n \rightarrow \infty} b_n = 0 = b$$

Thus

$$\lim_{n \rightarrow \infty} \frac{1}{n} \left[\left(\frac{1}{2}\right)^n + \dots \right] = a \cdot b = 0 \cdot 0 = \boxed{0} \quad \text{A}$$