Parameterize given surface S of cylinder $x^2+y^2=1$

Let $x = r\cos\theta$ $\quad y = r\sin\theta$ $\quad z = z$

$\boxed{0 \le \theta \le 2\pi}$ & $0 \le z \le 1+x$

$\boxed{0 \le z \le 1+\cos\theta}$

$\vec{r} = \cos\theta\,\hat{i} + (\sin\theta)\hat{j} + z\hat{k}$

$r_\theta = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j} + 0$

$r_z = 0 + 0 + \hat{k}$

$$\Rightarrow r_\theta \times r_z = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$

$$|r_\theta \times r_z| = \sqrt{\cos^2\theta + \sin^2\theta} = 1$$

$$\Rightarrow d\sigma = |r_\theta \times r_z|\, dz\, d\theta$$

Now

$$\iint_S 3z^2\, d\sigma = \int_0^{2\pi} \left(\int_{z=0}^{1+\cos\theta} 3z^2\, dz \right) d\theta$$

$$\Rightarrow \int_0^{2\pi} 3\left[\frac{z^3}{3}\right]_0^{1+\cos\theta} d\theta \Rightarrow \boxed{\int_0^{2\pi} (1+\cos\theta)^3\, d\theta}$$

option (a)