$$A = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

$A^{100} = ?$

$Ch_A(x) = x^3 - (\text{trac } A)x^2 + [(4-1) + (4-1) + (4-1)]x + 0$

$Ch_A(x) = x^3 - 6x^2 + 9x$

and $ch_A(x) = x^3 - 6x^2 + 9x = x(x^2 - 6x + 9)$

$Ch_A(x) = x(x^2 - 3x - 3x + 9)$

$Ch_A(x) = x(x(x-3) - 3(x-3))$

$Ch_A(x) = x(x-3)^2$

roots of $Ch_A(x)$ is $\underline{0, 3, 3}$

and let $A^{100} = aA + bA^2 + c$

Now $A^{100} = aA^2 + bA + c$ — (*)

put $x = A$

$x^{100} = ax^2 + bx + c$

$x = 0$

$0 = c$

Now

$x^{100} = ax^2 + bx$ — (1)

$x = 3$ is eigen value of A then $(3)^{100}$ is eigen value of A^{100}

$(3)^{100} = a(3)^2 + b \cdot 3$

$(3)^{99} = 3a + b$ — (2)

Now diff. (1) $100 \cdot x^{99} = 2ax + b$

put $x = 3$

$100(3)^{99} = 2a \cdot 3 + b$ — (3)

by eqn (2) and (3)

$(3)^{99} = 3a + \cancel{b}$

$100 \cdot (3)^{99} = 6a \cancel{+b}$

$3^{99}(1-100) = 3a \Rightarrow a = -\frac{99}{3} \cdot (3)^{99}$

$a = -33(3)^{99}$

and

Now put value of a in ②

$$(3)^{99} = 3 \times (-33)(3)^{99} + b$$

$$\underline{3^{99} \cdot 100 = b}$$

Now put value of a, b, c in eqn ⊛

$$\boxed{A^{100} = (-33)(3)^{99} A^2 + 3^{99} \cdot 100 \cdot A}$$

put value of A^2 and A the find A^{100}

$$\boxed{A^{100} = (3)^{99} \left[-33 A^2 + 100 A\right)}$$