$$\sum_{k=1}^{n} \frac{n}{n^2+k} = \frac{n}{n^2+1} + \frac{n}{n^2+2} + \cdots \frac{n}{n^2+n}$$

Now

$$\underbrace{\frac{n}{n^2+n} + \frac{n}{n^2+n} \cdots \frac{n}{n^2+n}}_{n\text{-times}} < \frac{n}{n^2+1} + \frac{n}{n^2+2} + \cdots \frac{n}{n^2+n} < \underbrace{\frac{n}{n^2+1} + \frac{n^2}{n^2+1} + \cdots \frac{n}{n^2+1}}_{n \text{ times}}$$

$$\frac{n \cdot n}{n^2+n} < \sum_{k=1}^{n} \frac{n}{n^2+k} < \frac{n \cdot n}{n^2+1}$$

$$\frac{n^2}{n^2+n} < \sum_{k=1}^{n} \frac{n}{n^2+k} < \frac{n^2}{n^2+1}$$

$$\frac{1}{1+\frac{1}{n}} < \sum_{k=1} \frac{n}{n^2+k} < \frac{1}{1+\frac{1}{n^2}}$$

$$\lim_{n\to\infty} \left(\frac{1}{1+\frac{1}{n}}\right) < \lim_{n\to\infty} \sum_{k=1} \frac{n}{n^2+k} < \lim_{n\to\infty} \frac{1}{1+\frac{1}{n^2}}$$

$$1 < \lim_{n\to\infty} \sum \frac{n}{n^2+k} < 1$$

$$\lim_{n\to\infty} \sum \frac{n}{n^2+k} = 1$$

Hence series is convergent.