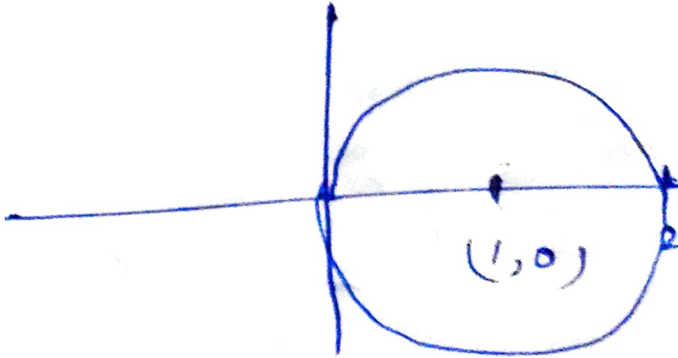
Green's theorem

$$\int_C P\,dx + Q\,dy = \iint \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx\, dy$$

where c is closed counterclockwise curve.

$$\oint_c -\frac{4}{3}xy^3 \cdot dx + x^4 \cdot dy$$



$$P = -\frac{4}{3}xy^3 \qquad Q = x^4$$

$$\frac{\partial P}{\partial y} = -\frac{4}{3} \times 3xy^2 = -4xy^2$$

$$\frac{\partial Q}{\partial x} = 4x^3$$

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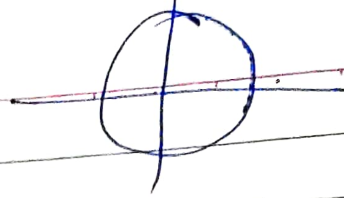
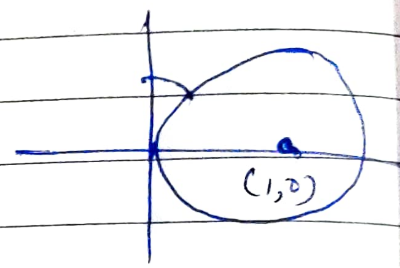


using greens theorem

$$\int_C -\frac{4}{3} xy^3 dx + x^4 dy = \iint (4x^3 + 4xy^2) \cdot dx\,dy \quad \text{--- (1)}$$

Convert integration in polar form for this first we convert eqn of curve in polar form



put $x = r\cos\theta$

$y = r\sin\theta$

then $(x-1)^2 + y^2 = 1$

$$(r\cos\theta - 1)^2 + (r\sin\theta)^2 = 1$$

$$r^2\cos^2\theta + 1 - 2r\cos\theta + r^2\sin^2\theta = 1$$

$$r^2 - 2r\cos\theta = 0$$

$$\boxed{r = 2\cos\theta} \quad \text{eqn of circle}$$

Now

$$4\iint x(x^2+y^2)\,dx\,dy$$

$$= 4\int_0^{2\cos\theta}\int_{-\pi/2}^{\pi/2} r\cos\theta\,(r^2) \cdot dr\,d\theta$$

$\theta = -\frac{\pi}{2}$ to $\theta = \frac{\pi}{2}$ & $r = 0$ to $r = 2\cos\theta$

$$= 4\int_0^{2\cos\theta}\int_{-\pi/2}^{\pi/2} r^3 \cdot \cos\theta\,dr\,d\theta$$

$$= 4\int_{-\pi/2}^{\pi/2}\left[\frac{r^4}{4}\right]_0^{2\cos\theta} \cos\theta\,d\theta$$

$\frac{4\times 2}{4}\int_0^{\pi/2} (2)^4 \cdot \cos^4\theta \, d\theta$

$= (2)^5 \int_0^{\pi/2} \cos^4\theta \cdot d\theta$ — (2)

using gama function

$\int_0^{\pi/2} \sin^m\theta \cos^n\theta \, d\theta = \frac{\Gamma\frac{m+1}{2} \Gamma\frac{m+1}{2}}{2\Gamma\frac{m+n+2}{2}}$

$\int_0^{\pi/2} \cos^4\theta \sin^0\theta \cdot d\theta = \frac{\Gamma\frac{4+1}{2} \Gamma\frac{0+1}{2}}{2\Gamma\frac{4+0+2}{2}}$

$= \frac{\Gamma\frac{5}{2} \Gamma\frac{1}{2}}{2\Gamma 3}$

$= \frac{\frac{3}{2}\times\frac{1}{2}\Gamma\frac{1}{2} \Gamma\frac{1}{2}}{2\times 2!}$ — (3)

Now put value from (3) to (2)

$(2)^5 \times \frac{3 \cdot \pi}{(2)^4}$ since $\Gamma\frac{1}{2} = \sqrt{\pi}$

$2 \times 3\pi$

6π