dx

since $o(G) = 24$

$\Rightarrow \quad a^{24} = e.$

possible order of elements of group G is =

divisior's of 24

1, 2, 3, 4, 6, 8, 12, 24

since given that $a^8 \neq a^{12} \neq e$

if $o(a) = 1 \Rightarrow a^1 = e \Rightarrow a^8 = e \quad a^{12} = e$ But Not possib.

Here $o(a) \neq 1$

$o(a) = 2 \Rightarrow a^2 = e \Rightarrow a^8 = e, a^{12} = e$ Not possible

$o(a) = 3 \Rightarrow a^3 = e \Rightarrow a^{12} = e \rightarrow$ Not possible

If $o(a) = 4 \Rightarrow a^4 = e \Rightarrow a^8 = e$ & Not possible
$a^{12} = e$

If $o(a) = 6 \Rightarrow a^6 = e \Rightarrow a^{12} = e$ Not possible

and $o(a) = 8 \Rightarrow a^8 = e$ Not possible

and $o(a) = 12 \Rightarrow a^{12} = e$ Not possible

Here $\boxed{o(a) = 24}$

$a^{24} = e$

(b) $S = (1\,3\,5\,7\,11)(2\,4\,6) \in S_{11}$

$\quad\quad I_1 \quad\quad I_2$

$S = I_1 \cdot I_2$

$o(I_1) = 5$

$o(I_2) = 3$

Smallest positive No. or order of S is

$o(S) = \text{L.C.M}(o(I_1), o(I_2))$

$o(S) = \text{L.C.M}(5, 3) = 15$

$\Rightarrow$ $o(S) = 15 \Rightarrow S^{15} = I$

$S^{30} = I$

Now $S^{31} = S$

$S^{32} = S^2$

$\vdots$

$S^{37} = S^7.$

Hence smallest No. is $\underline{\underline{7}}$.