

Given that $a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ this polynomial has complex root with negative real part.

Now. $a_n \frac{d^n u}{dt^n} + a_{n-1} \frac{d^{n-1} u}{dt^{n-1}} + \dots + a_1 \frac{du}{dt} + a_0 u = 0$

Solution of above diff. eqⁿ is such as

$$\frac{d}{dt} \equiv D$$

$$[a_n D^n + a_{n-1} D^{n-1} + \dots + a_1 D + a_0] u = 0$$

A.E. $a_n m^n + a_{n-1} m^{n-1} + \dots + a_1 m + a_0 = 0$

Since given that roots of this polynomial has complex roots ~~then~~ ~~set~~ such as

$$-\alpha_1 \pm i\beta_1, \alpha_2 \pm i\beta_2, \dots, \alpha_p \pm i\beta_p$$

and solution corresponding to this

Ans $u(x) = e^{-\alpha_1 x} [\cos \beta_1 x + i \sin \beta_1 x] + \dots$

$$e^{-\alpha_p x} [\cos \beta_p x + i \sin \beta_p x]$$

$$\lim_{x \rightarrow \infty} u(x) = e^{-\infty} (\quad) + \dots + e^{-\infty} (\quad)$$

$$\boxed{\lim_{x \rightarrow \infty} u(x) = 0}$$