Ans $f : \mathbb{C} \to \mathbb{C}$ entire function

$g : \mathbb{C} \to \mathbb{C}$ entire function

s.t. $g(z) = f(z) - f(z+1)$

Using Identity theorem

(a) $f(\frac{1}{n}) = 0 \Rightarrow f$ is constant (True)

$A = \frac{1}{n}$

A' = set of limit point $= \{0\}$

$A' \cap C = \{0\} \neq \phi$

Hence $f(z) \equiv 0$ for each $z \in C$

(c) $f(\frac{1}{n}) = f(\frac{1}{n}+1)$ $\forall n \in N$ then g is a constant function (True)

since $A = \frac{1}{n}$

$A' = 0$

$A' \cap C = \{0\} \neq \phi$

Hence $f(\frac{1}{n}) \equiv f(\frac{1}{n}+1)$ $\forall$ C

$g(\frac{1}{n}) = f(\frac{1}{n}) - f(\frac{1}{n}+1)$

$= 0$ $\forall C$

$\Rightarrow \boxed{g(z) \equiv 0}$

We can't apply Identity theorem remaining two ~~optati~~ (b) and (d)

Identity theorem

First form :-

$f : D \xrightarrow{\text{analytic}} C$

$f(z) = c \quad \forall \; z \in A \subseteq D$

$A' \cap D \neq \phi \Rightarrow f(z) = c \quad \forall \; z \in D$

Second form :→

$f, g : D \xrightarrow{\text{analytic}} C$

$f(z) = g(z) \quad \forall \; z \in A \subseteq D$

$A' \cap D \neq \phi \Rightarrow f(z) = g(z) \quad \forall \; z \in D$